Sparkle Revisited:
Proving Tight Adaptive Security of a Simple Schnorr Threshold Scheme

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NIST Workshop on Multi-Party Threshold Schemes

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"Revisiting the Security of Sparkle"
on ePrint soon…

Threshold Signatures

- t-out-of-n Threshold Signatures (TS):
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- Security models: static vs. adaptive corruptions
- Schnorr signatures:
  - Standardized & widely deployed (e.g., EdDSA, Taproot)
  - Schnorr TS: “out-of-the-box” compatibility with plain Schnorr verification

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- These negative results identify proof deficiencies, not practical attacks!

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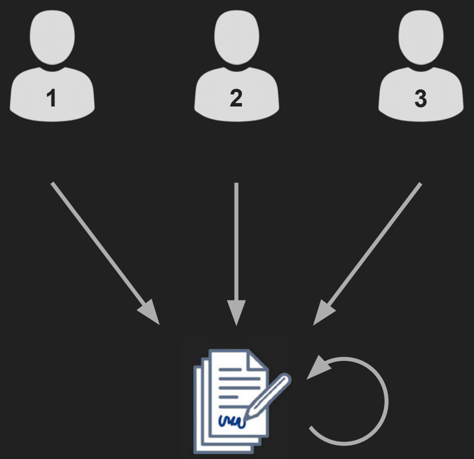
Our Question

Can the original Sparkle scheme be proved—statically or even adaptively—secure?

Goal 1: Full Adaptive Security

- Static corruption model



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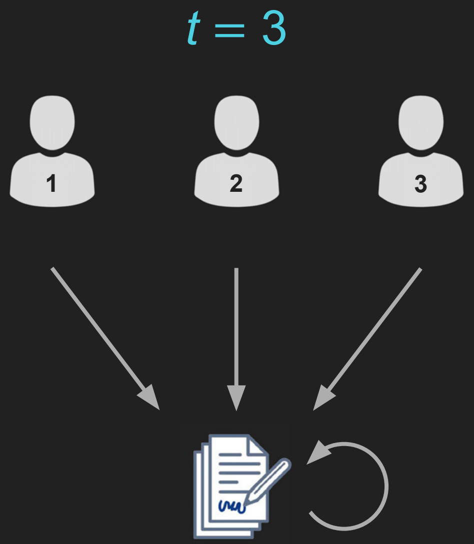

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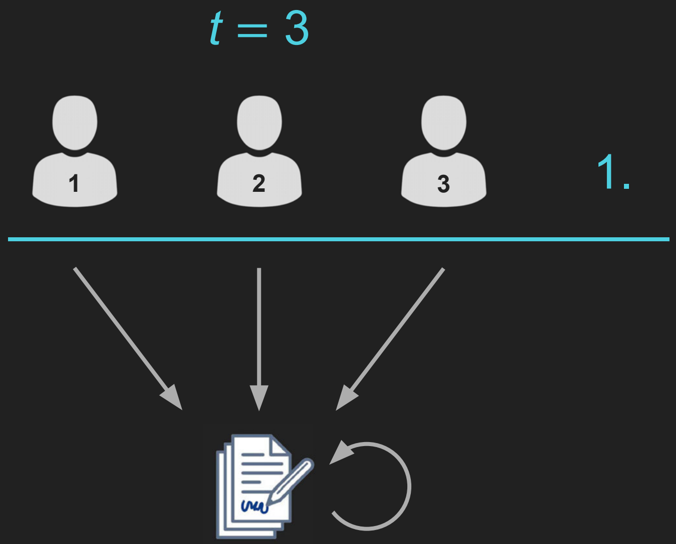

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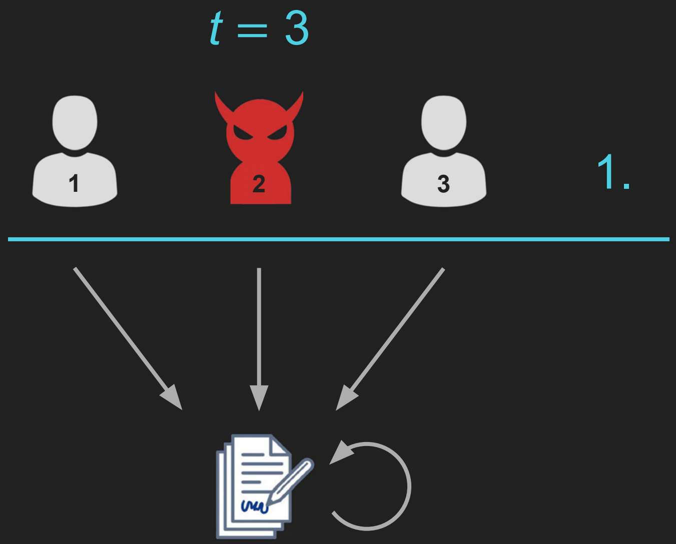

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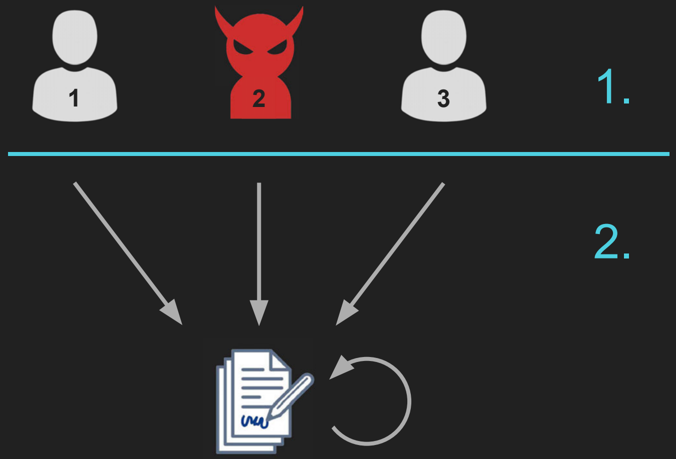

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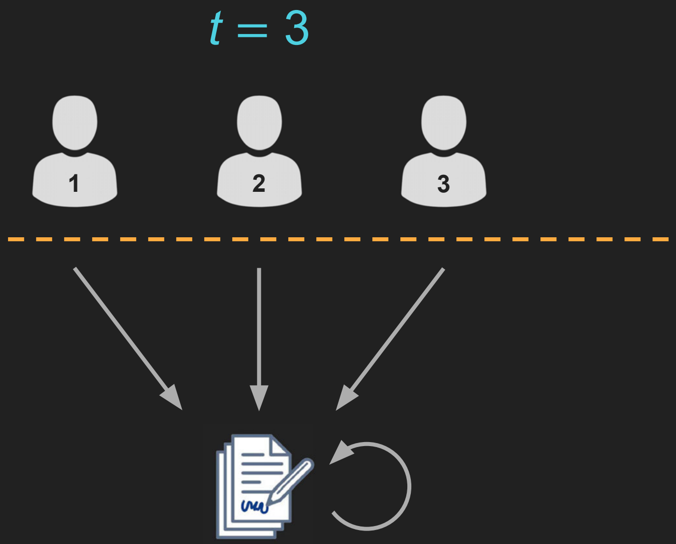

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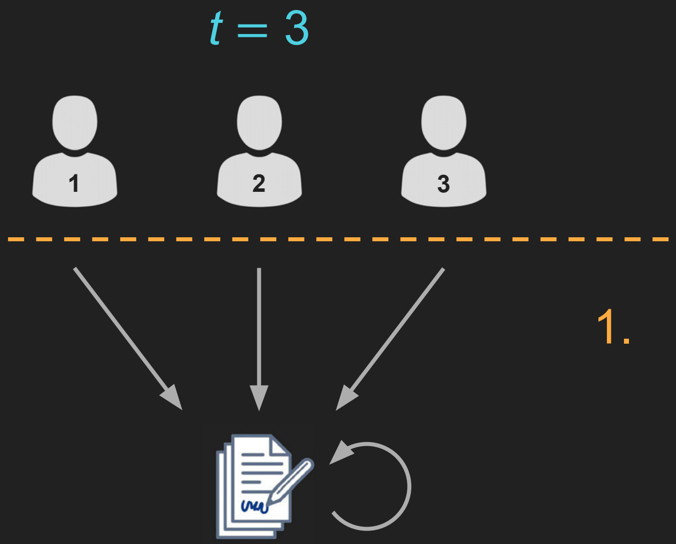

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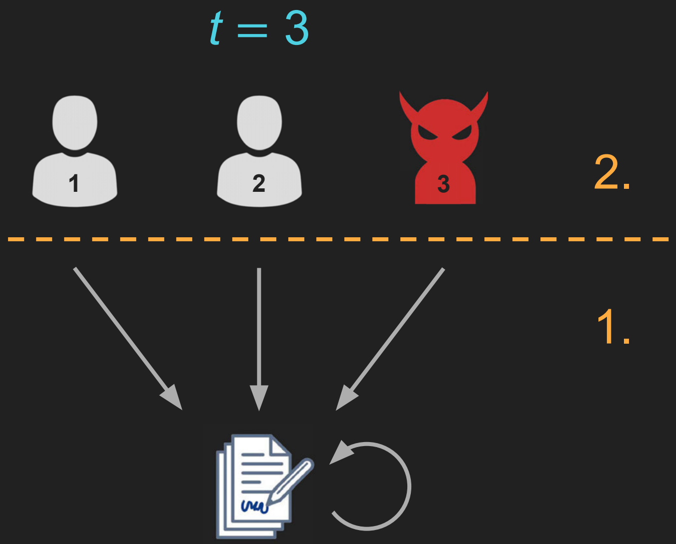

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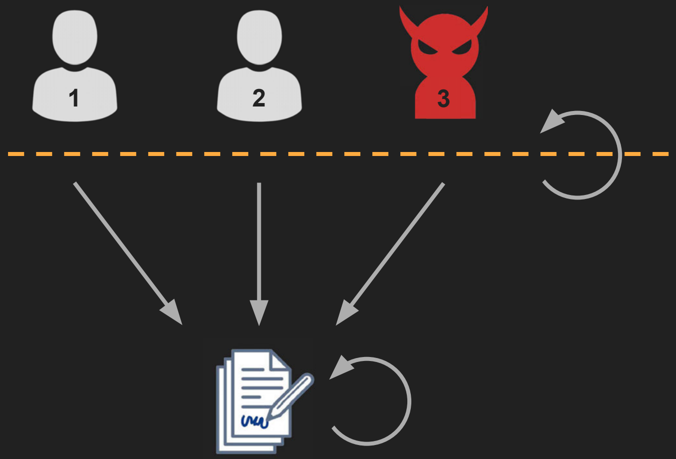

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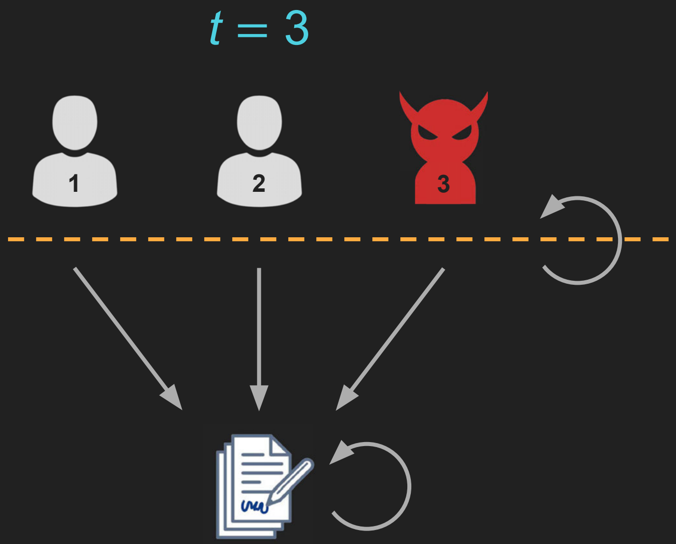

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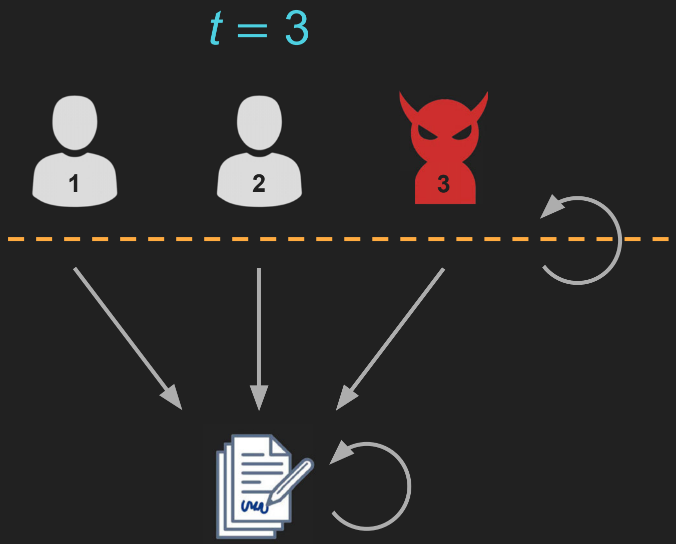

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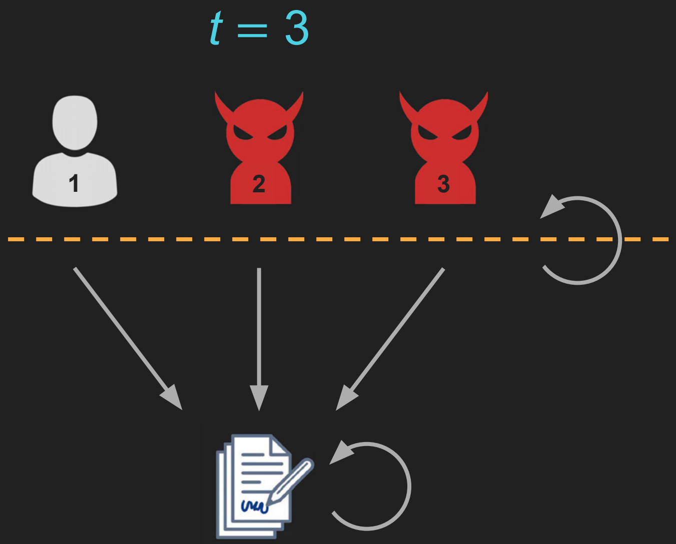

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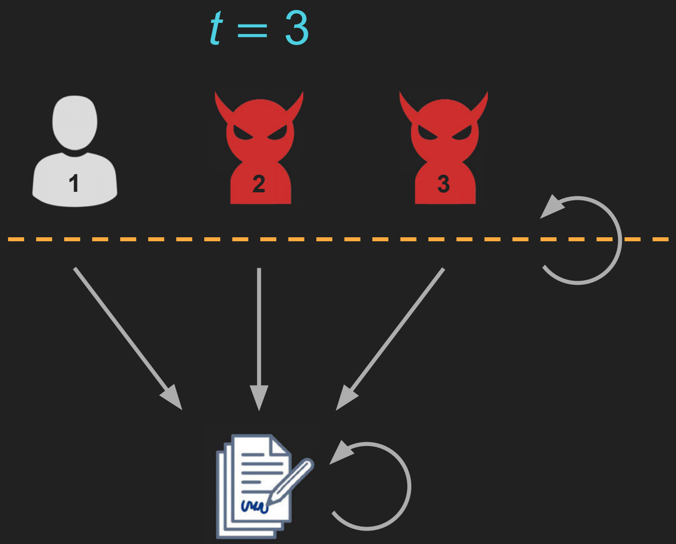

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  - E.g. FROST and its variants and the (flawed) full adp. security proof of Sparkle+
- Our goal is to avoid additional idealized models

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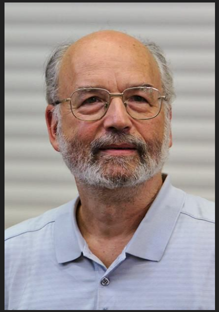

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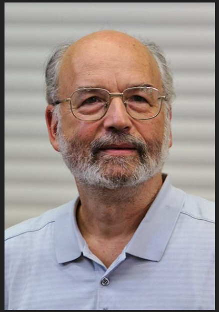

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Best known attack:
break DL, which takes time $O(\sqrt{|\mathbb{G}|})$

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Theoreticians

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[PS96] in **ROM**:

$$\mathsf{Adv}^{\text{euf-cma}}_{\mathsf{Sch}[\mathbb{G}]} \le q_h \cdot \sqrt{\mathsf{Adv}^{\text{dl}}_{\mathbb{G}}} + \dots$$

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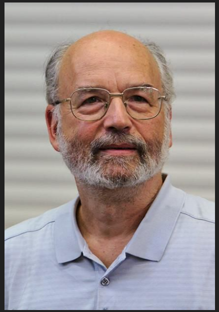

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2. Introduce new assumption VCDL $\Longrightarrow$ tight full adaptive security of Sparkle
3. Justify VCDL: reduce VCDL to necessary assumption LDVR [CKK+25] when idealizing the group

Comparison with Selected Schemes

Scheme	Rounds	Model	Comm. / Signer	Comp. / Signer
FROST	2	ROM+AGM	$2\mathbb{G} + \mathbb{Z}_p$	3 Exp
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Gargos	3	ROM	$2\mathbb{G} + 7\mathbb{Z}_p$	8 Exp + NIZK.P + t NIZK.V

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- Final Schnorr signature: $(R, z) = (\prod R_i, \sum z_i)$

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  - Fixed in Sparkle+ by having parties sign their local views

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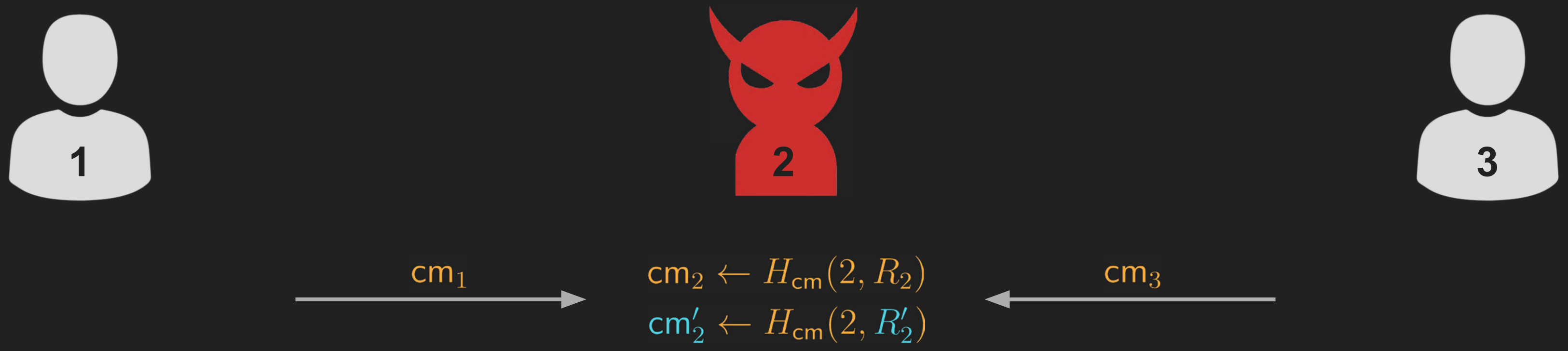

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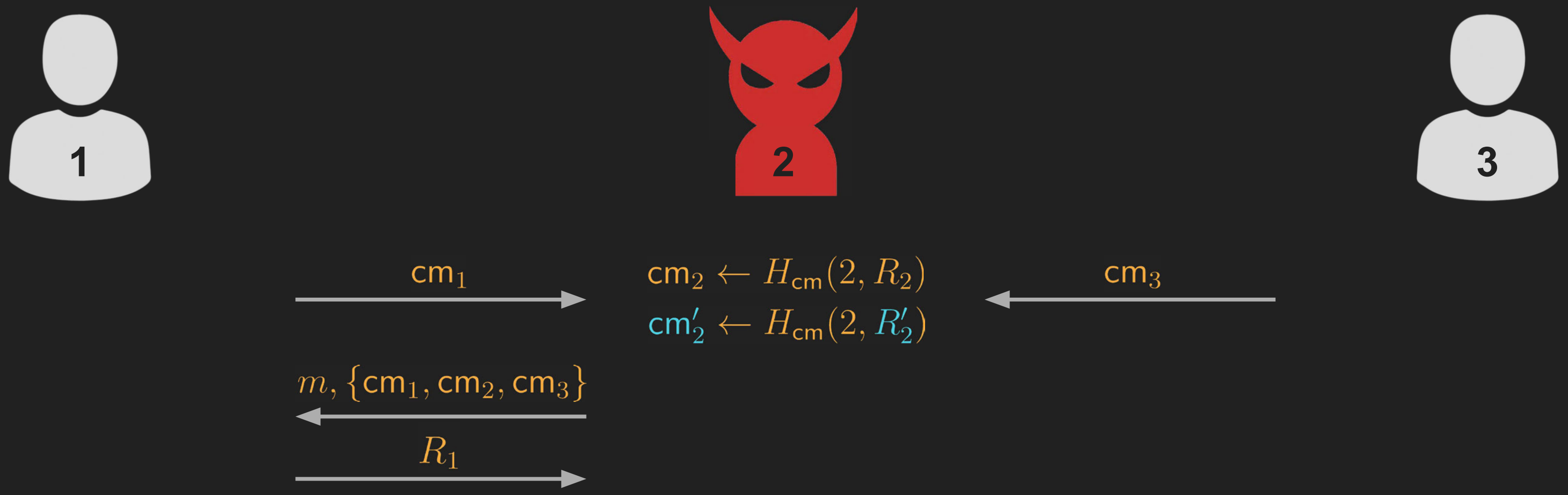

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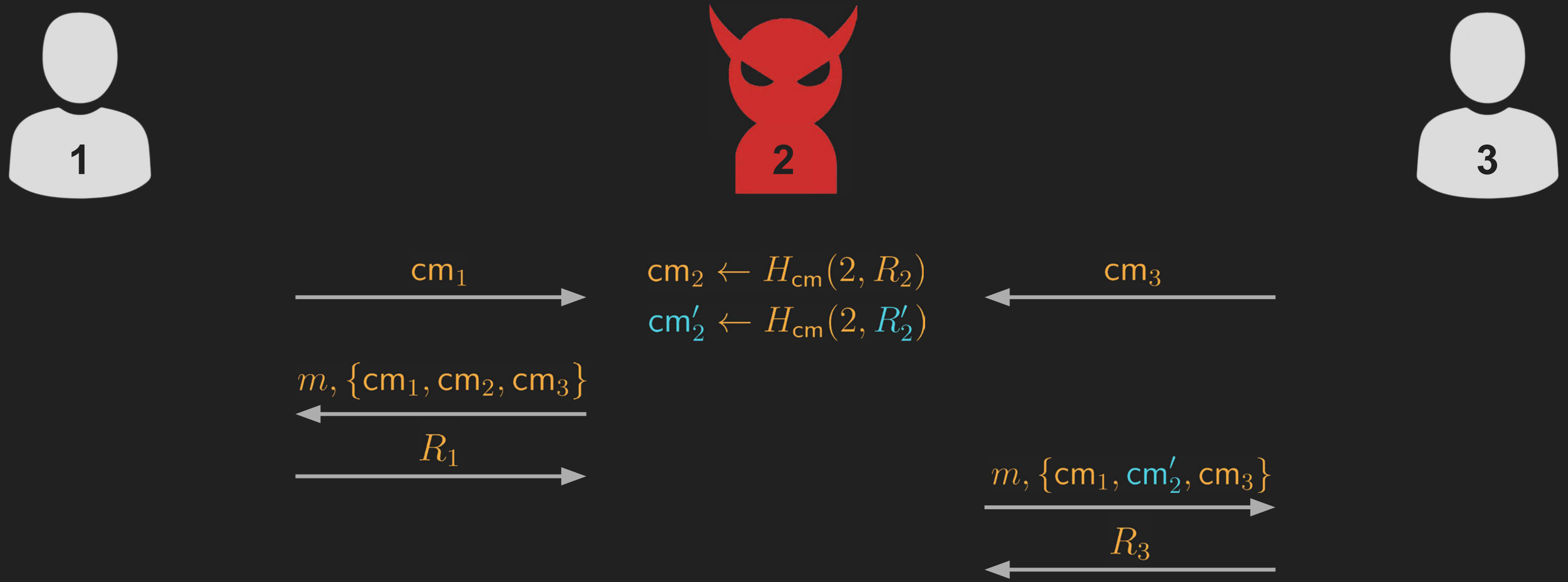

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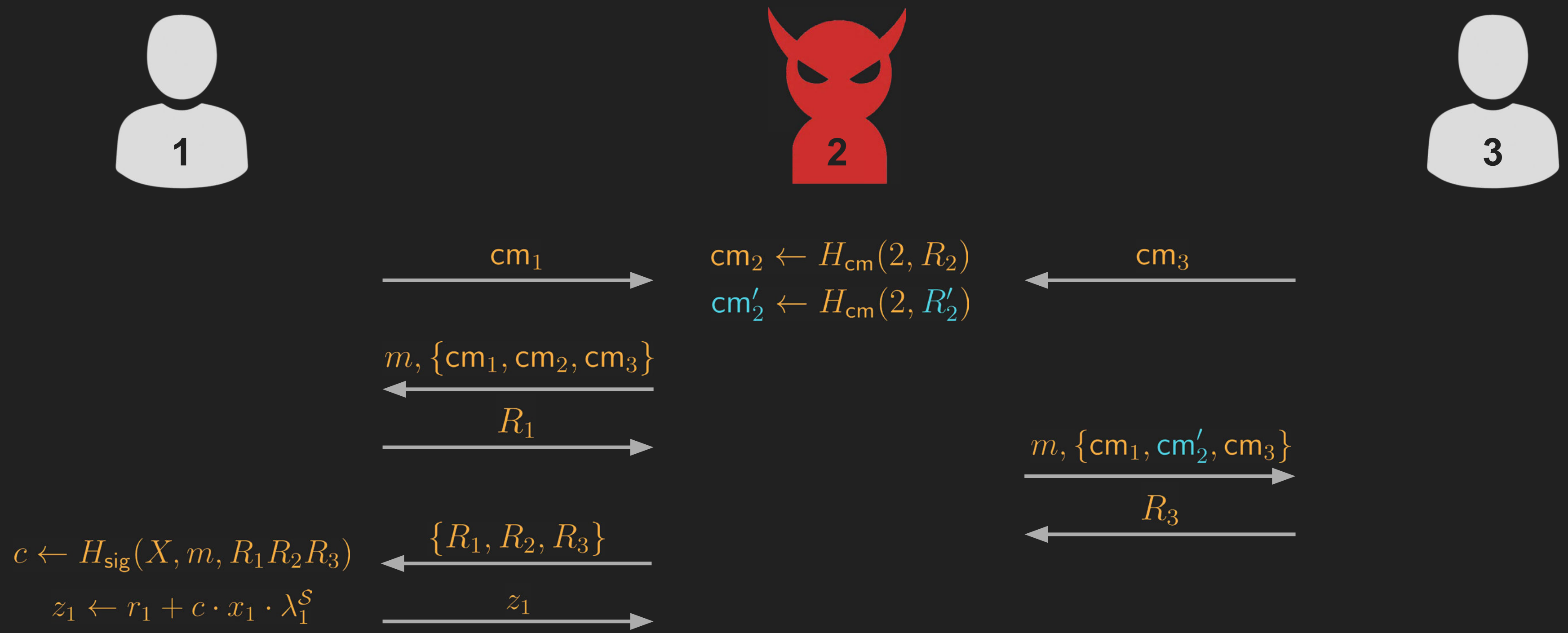

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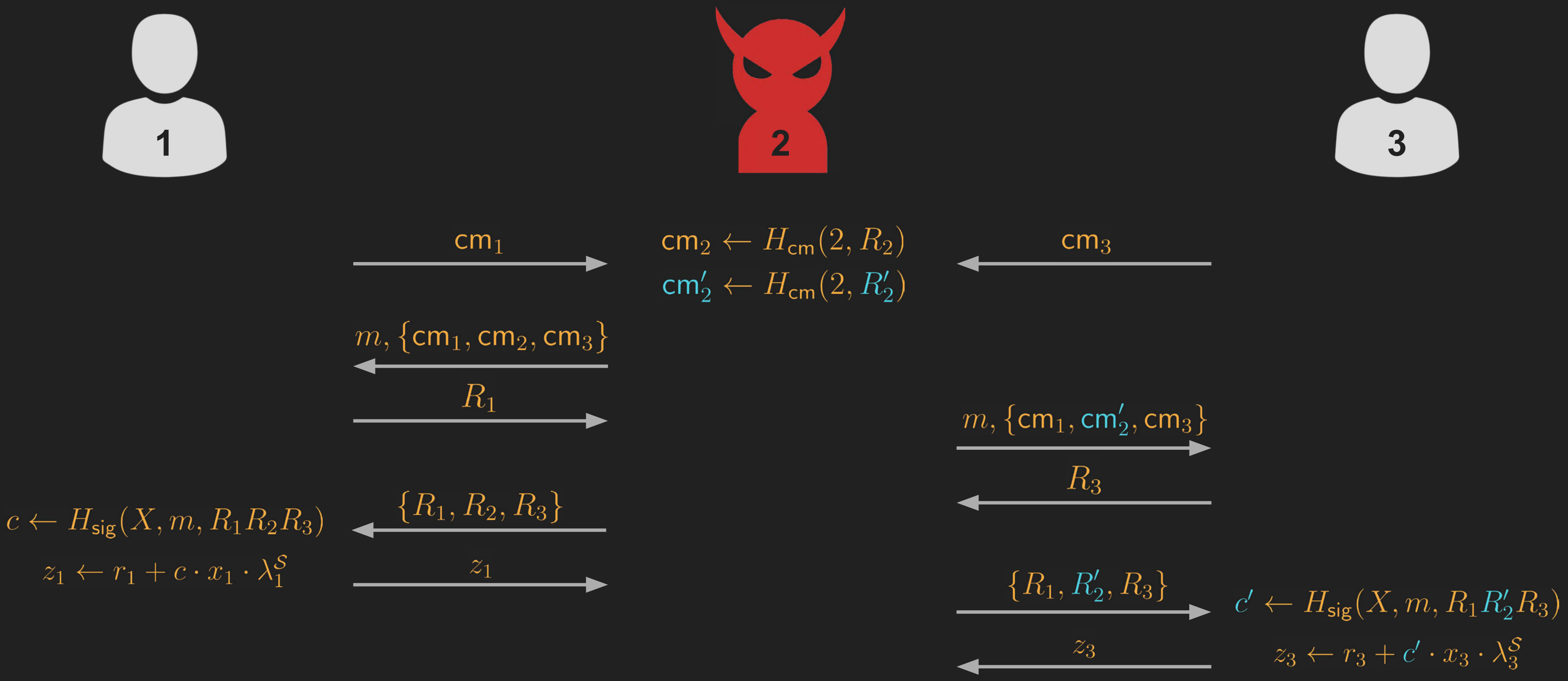

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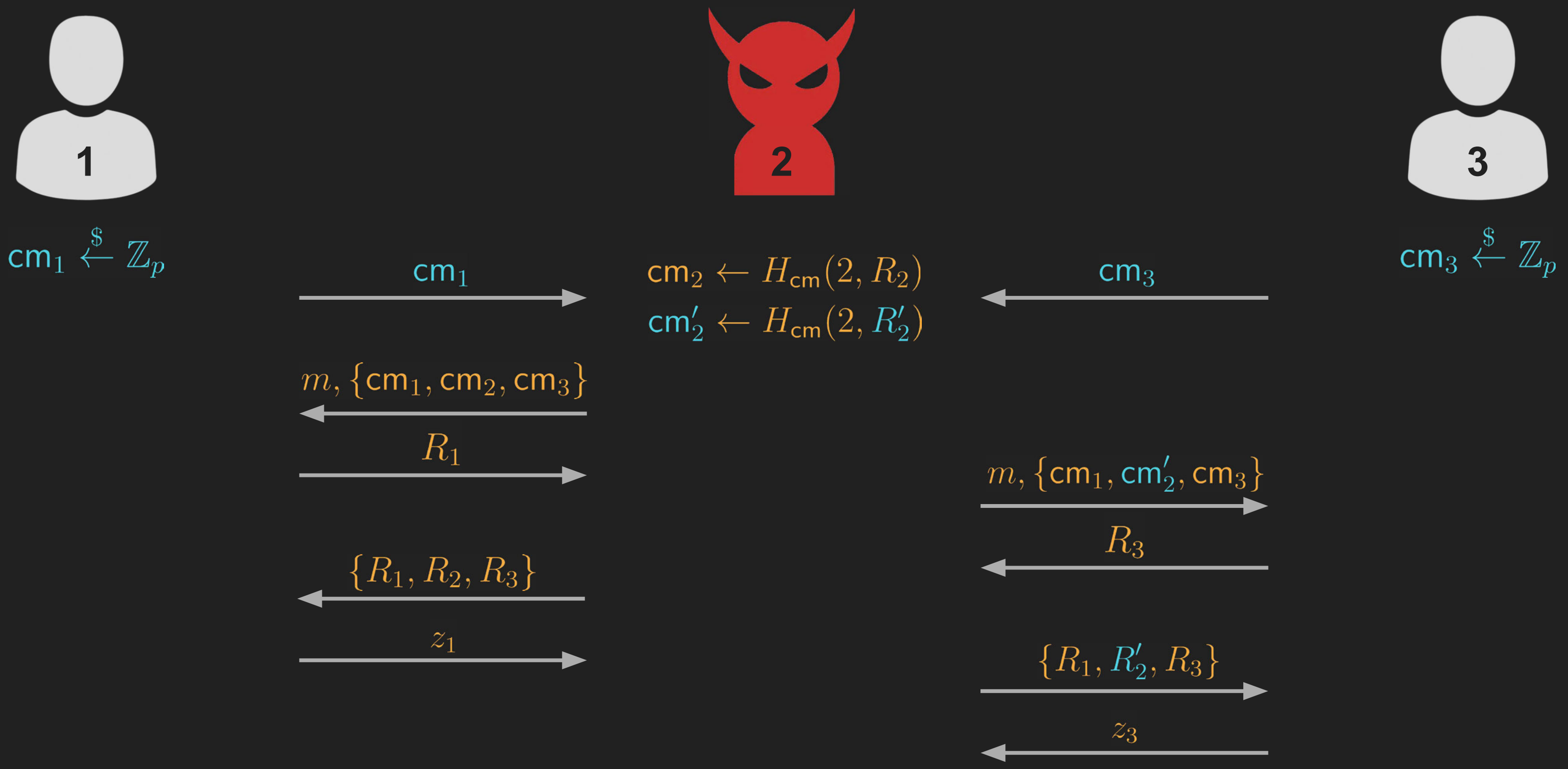

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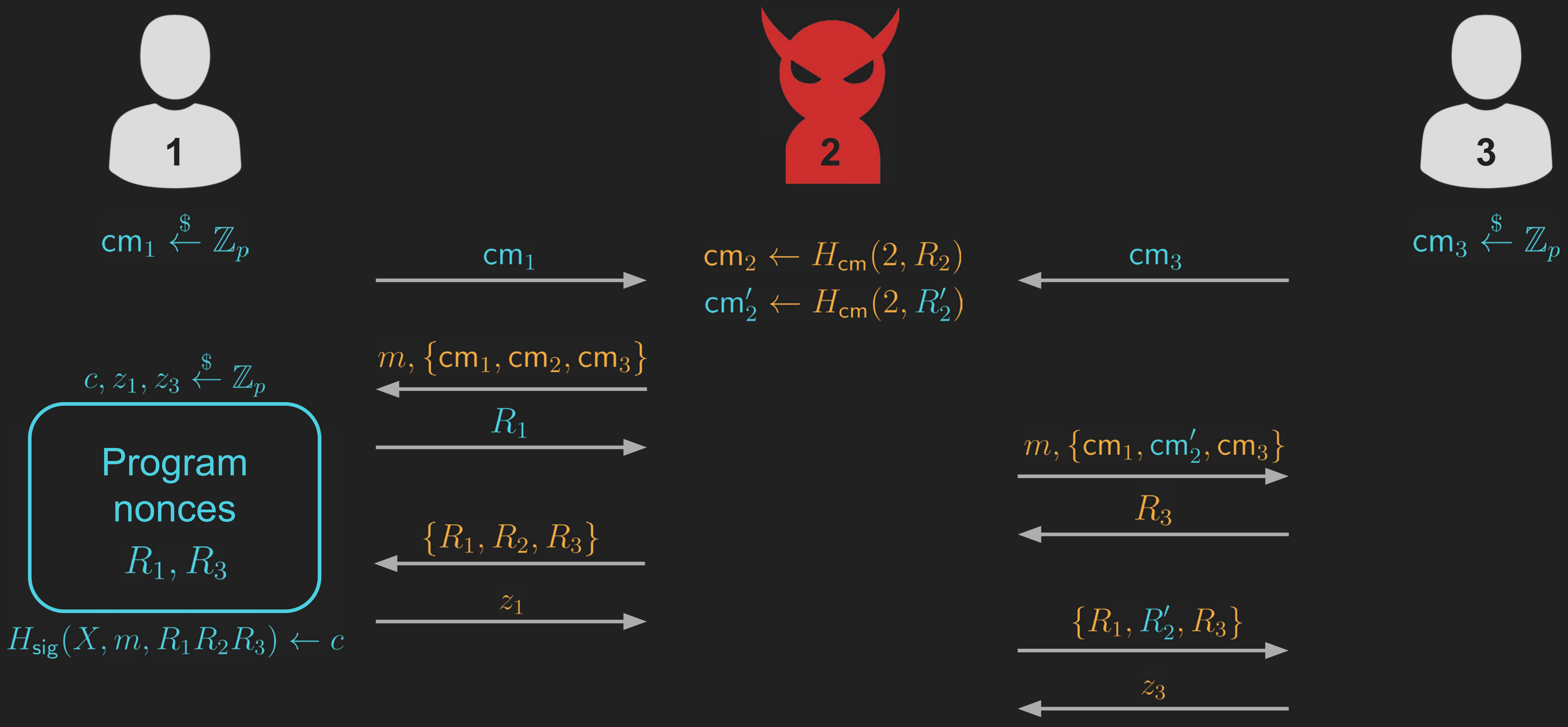
Counterexample: Original Simulation Strategy

1		2		3
$\mathsf{cm}_1 \xleftarrow{\$} \mathbb{Z}_p$	$\mathsf{cm}_1 \rightarrow$	$\mathsf{cm}_2 \leftarrow H_{\mathsf{cm}}(2, R_2)$ $\mathsf{cm}'_2 \leftarrow H_{\mathsf{cm}}(2, R'_2)$	$\leftarrow \mathsf{cm}_3$	$\mathsf{cm}_3 \xleftarrow{\$} \mathbb{Z}_p$
$c, z_1, z_3 \xleftarrow{\$} \mathbb{Z}_p$	$\leftarrow m, \{\mathsf{cm}_1, \mathsf{cm}_2, \mathsf{cm}_3\}$			
$R_1 \leftarrow g^{z_1} \cdot X_1^{-c \cdot \lambda_1^S}$ $H_{\mathsf{cm}}(1, R_1) \leftarrow \mathsf{cm}_1$	$R_1 \rightarrow$		$m, \{\mathsf{cm}_1, \mathsf{cm}'_2, \mathsf{cm}_3\} \rightarrow$	
			$\leftarrow R_3$	
$R_3 \leftarrow g^{z_3} \cdot X_3^{-c \cdot \lambda_3^S}$ $H_{\mathsf{cm}}(3, R_3) \leftarrow \mathsf{cm}_3$	$\leftarrow \{R_1, R_2, R_3\}$			
$H_{\mathsf{sig}}(X, m, R_1 R_2 R_3) \leftarrow c$	$z_1 \rightarrow$		$\{R_1, R'_2, R_3\} \rightarrow$	
			$\leftarrow z_3$	

Counterexample: Original Simulation Strategy



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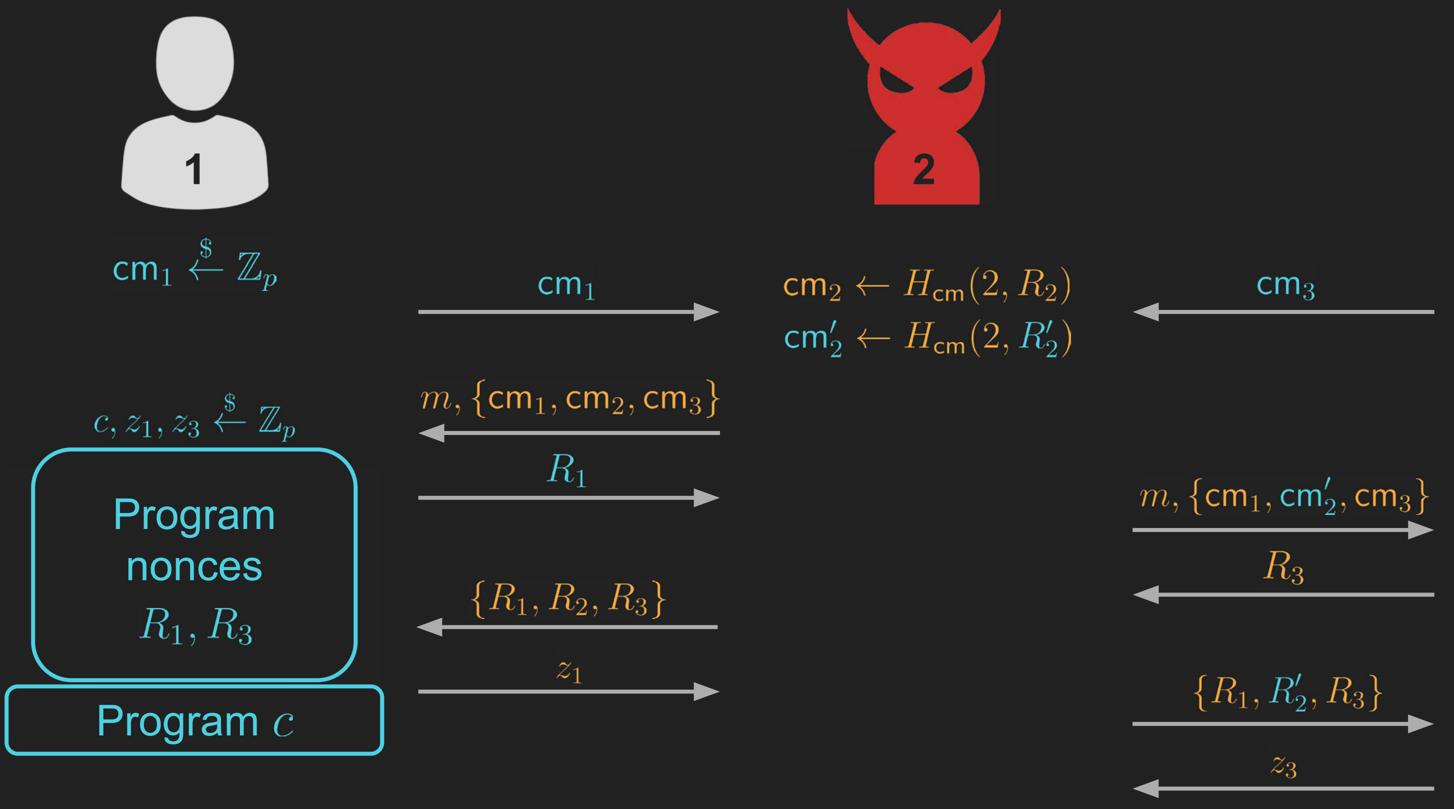

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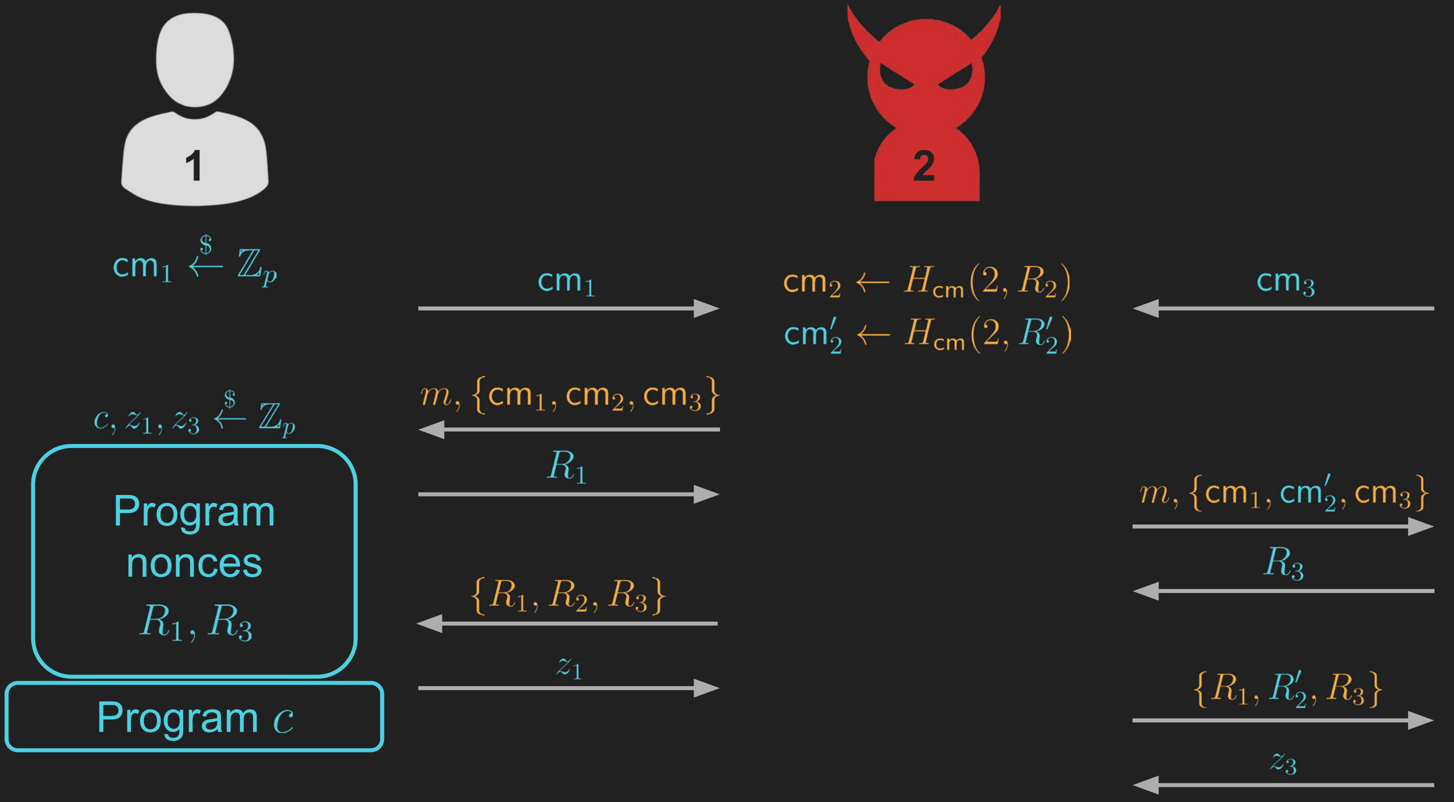

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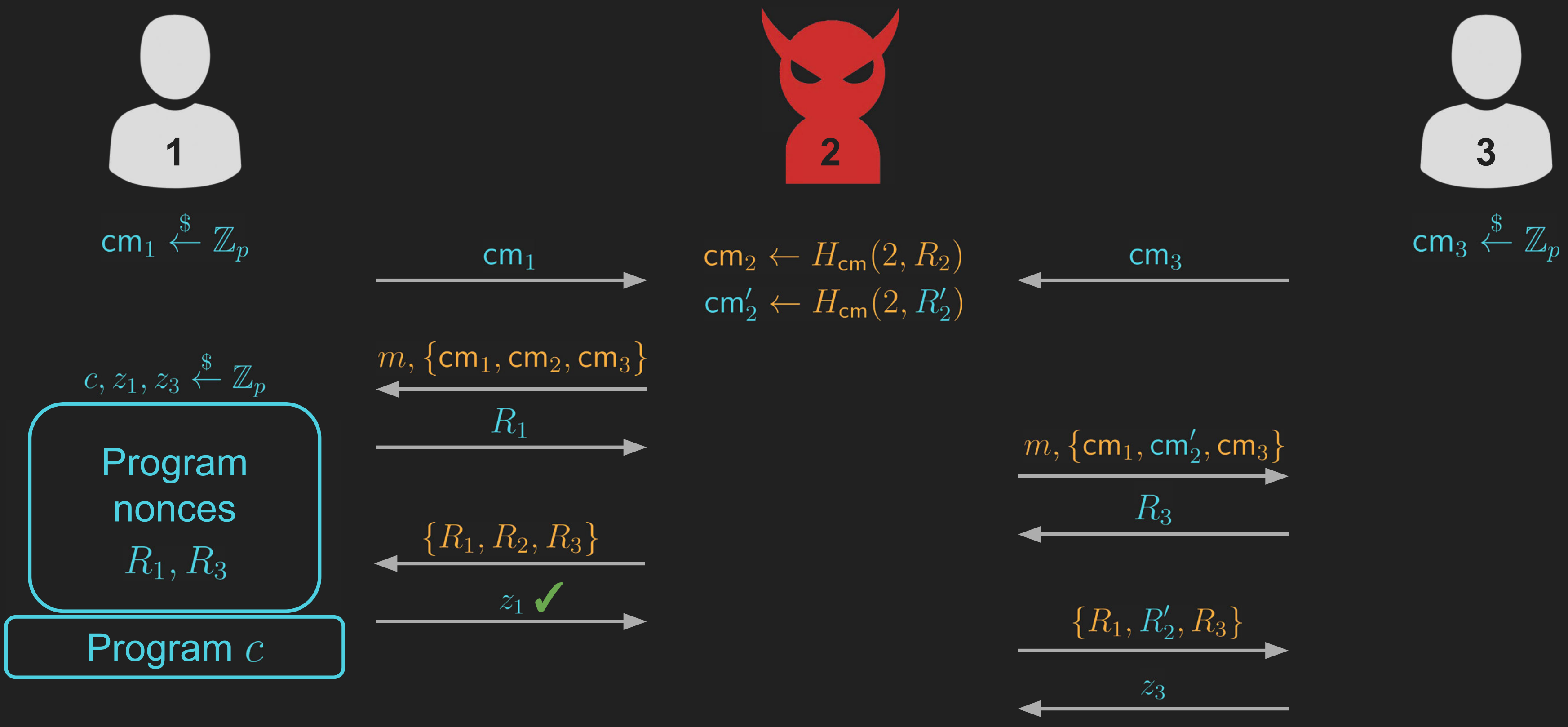

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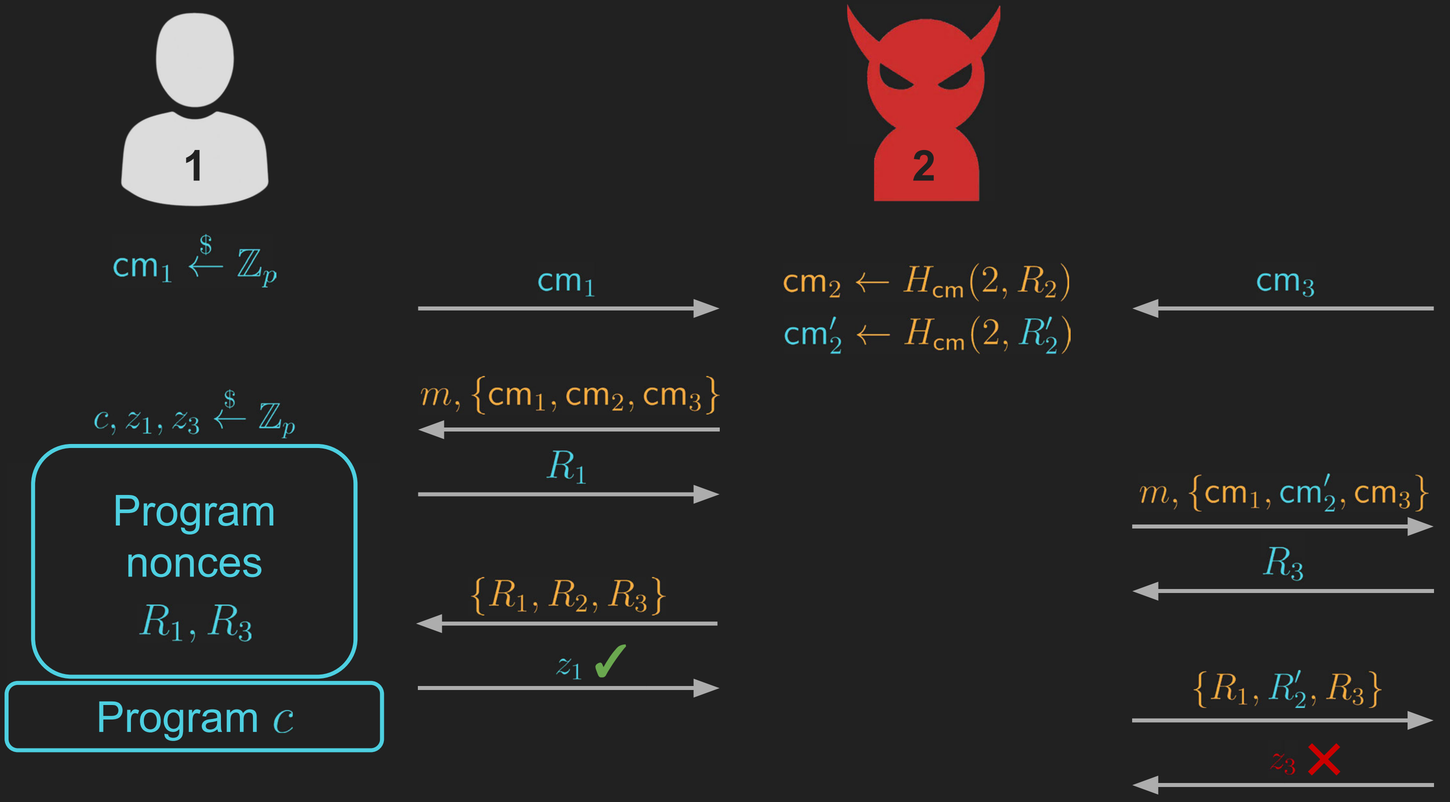

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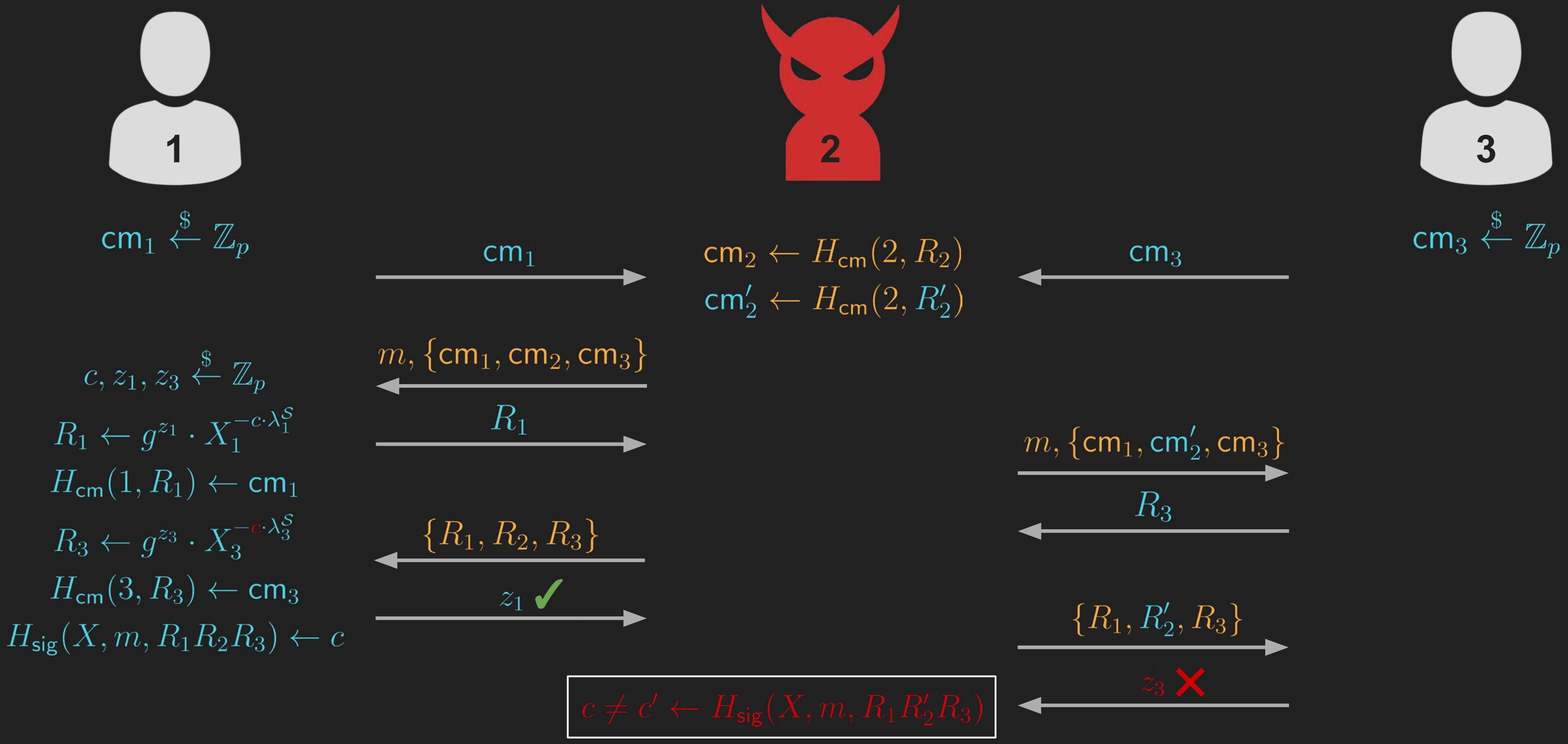

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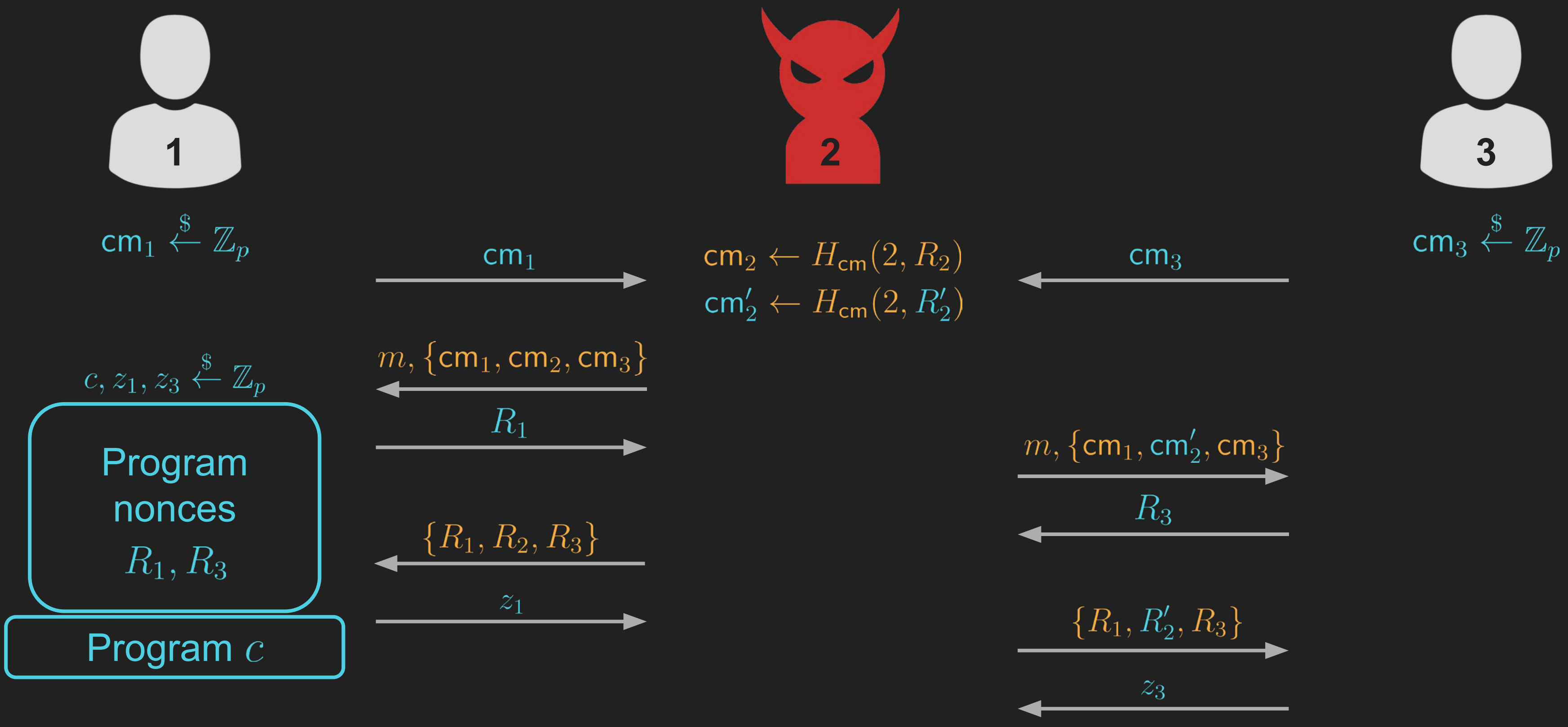

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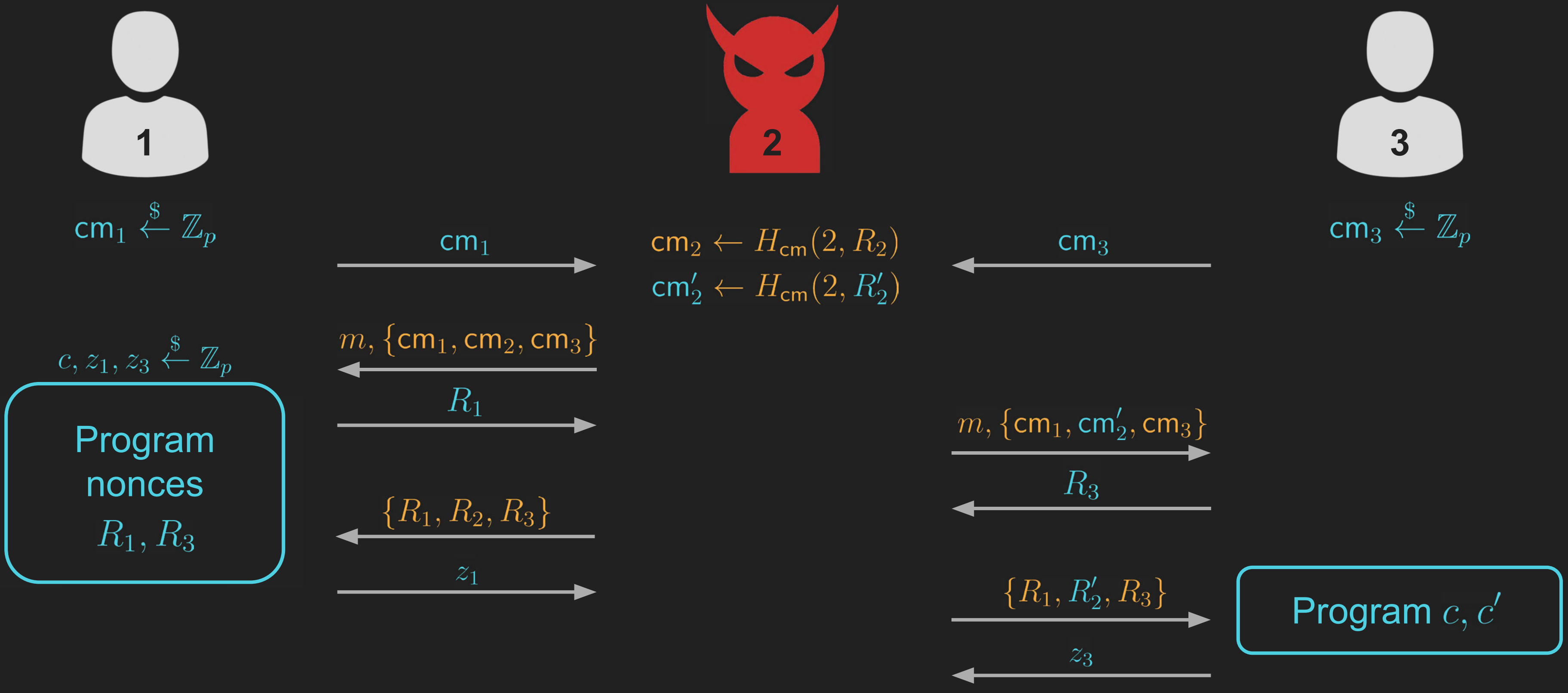

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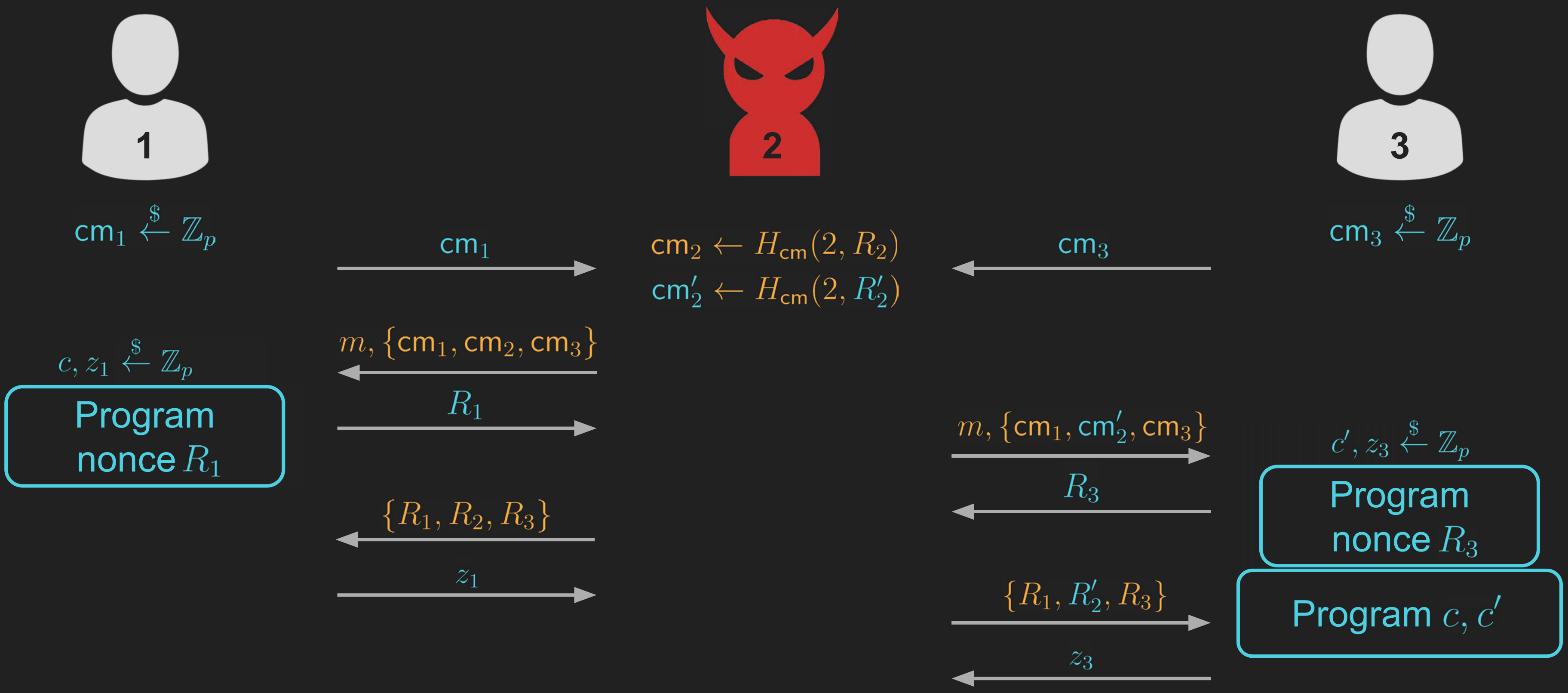

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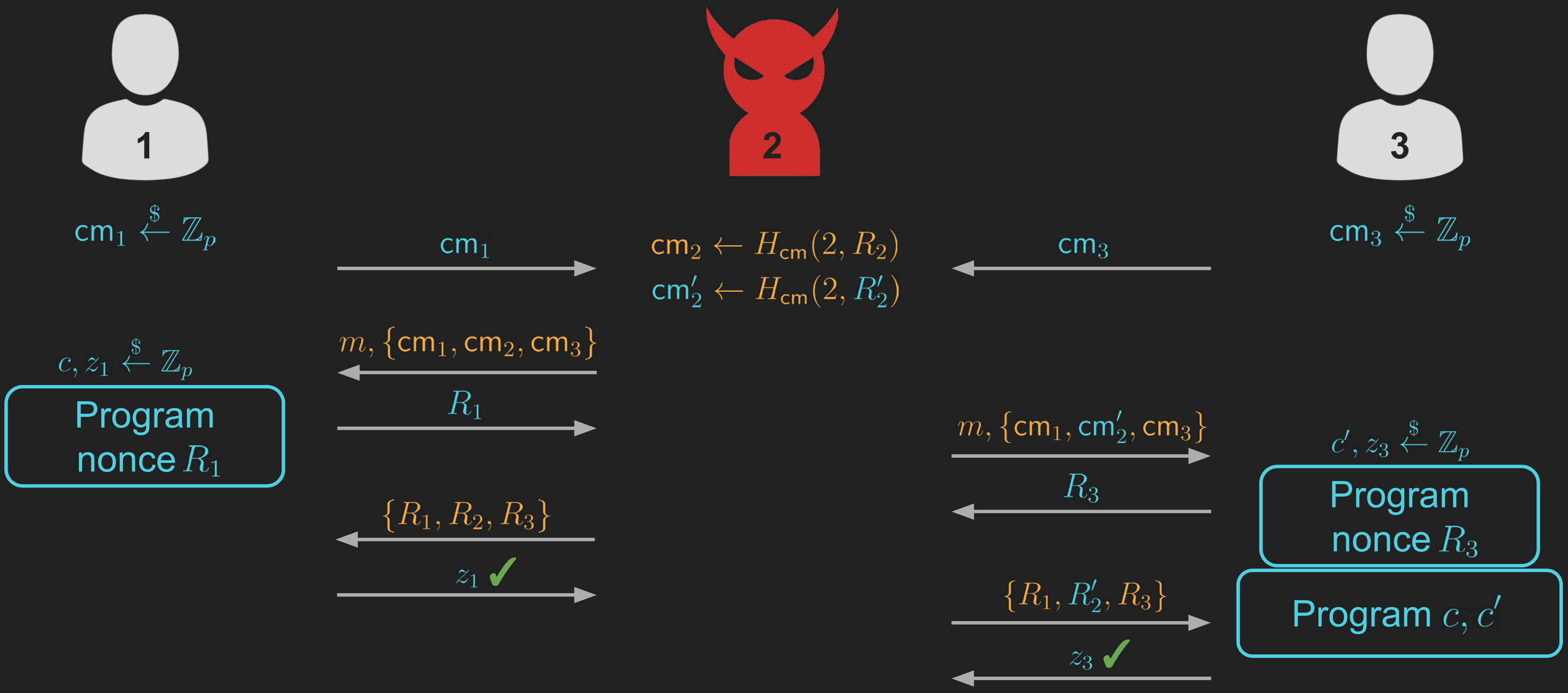

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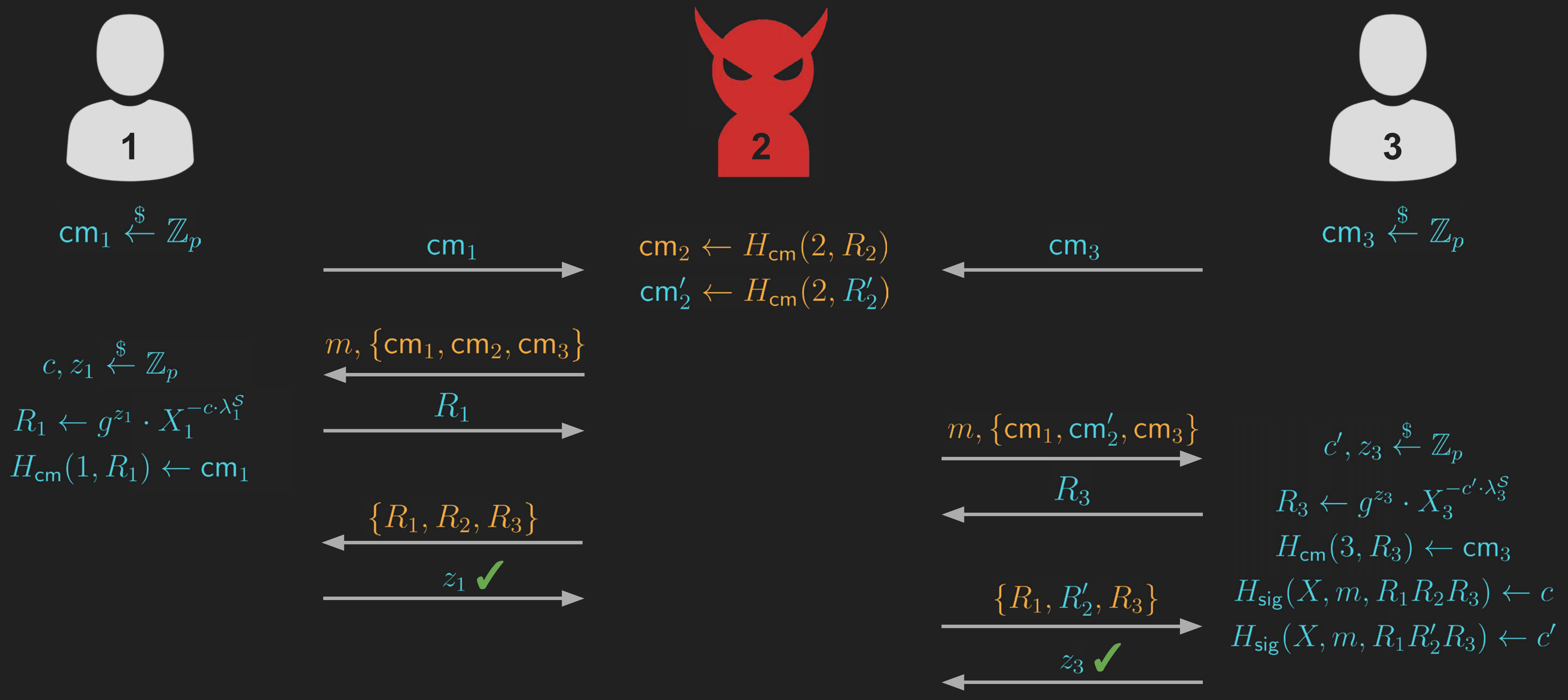

Problem 2:
Avoiding Rewinding

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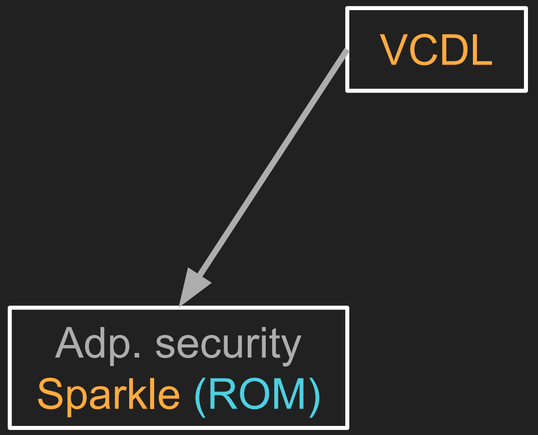
- We strengthen CDL to interactive variant: “Vandermonde” CDL (VCDL)

Main Results

- Tight proof of full adaptive security of Sparkle under VCDL in the ROM



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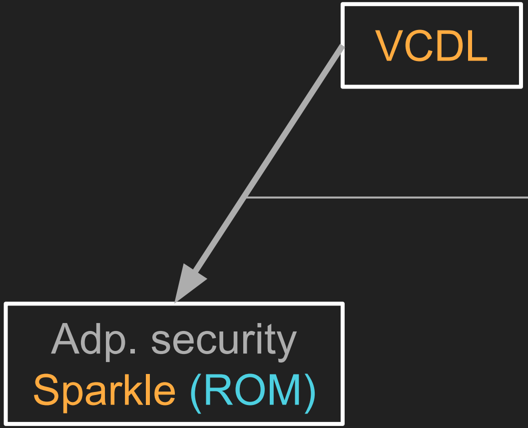

Theorem (in ROM):

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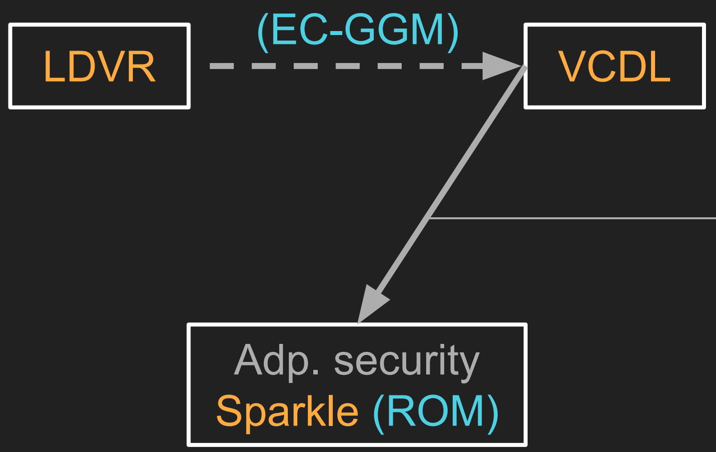

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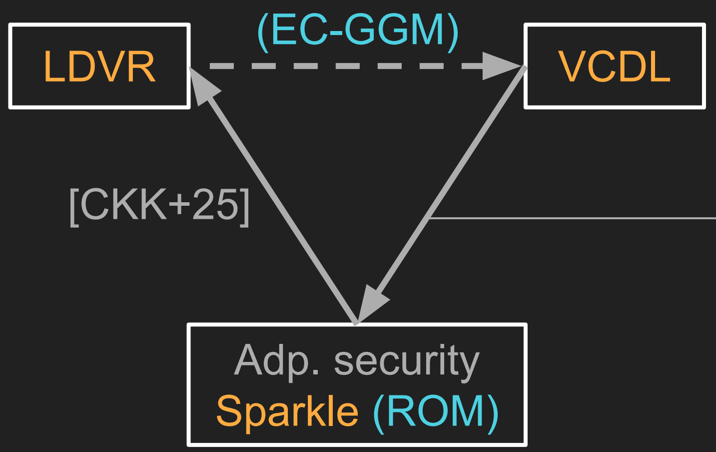

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More in the Full Paper…

“Revisiting the Security of Sparkle”

on ePrint soon…

Thanks!

Questions?

References

[BLT+24] Renas Bacho, Julian Loss, Stefano Tessaro, Benedikt Wagner, and Chenzhi Zhu. Twinkle: Threshold Signatures from DDH with Full Adaptive Security. In Advances in Cryptology – EUROCRYPT 2024, volume 14651 of LNCS, pages 429–459. Springer, 2024. https://doi.org/10.1007/978-3-031-58716-0_15.

[CFOS25] Gavin Cho, Georg Fuchsbauer, Adam O'Neill, and Marek Sefranek. Schnorr Signatures are Tightly Secure in the ROM Under a Non-interactive Assumption. In Advances in Cryptology – CRYPTO 2025, volume 16005 of LNCS, pages 223–255. Springer, 2025. https://doi.org/10.1007/978-3-032-01887-8_8.

[CKK+25] Elizabeth Crites, Jonathan Katz, Chelsea Komlo, Stefano Tessaro, and Chenzhi Zhu. On the Adaptive Security of FROST. In Advances in Cryptology – CRYPTO 2025, volume 16005 of LNCS, pages 480–511. Springer, 2025. https://doi.org/10.1007/978-3-032-01887-8_16.

[CKM23] Elizabeth Crites, Chelsea Komlo, and Mary Maller. Fully Adaptive Schnorr Threshold Signatures. In Advances in Cryptology – CRYPTO 2023, volume 14081 of LNCS, pages 678–709. Springer, 2023. https://doi.org/10.1007/978-3-031-38557-5_22.

[CS25] Elizabeth Crites and Alistair Stewart. A Plausible Attack on the Adaptive Security of Threshold Schnorr Signatures. In Advances in Cryptology – CRYPTO 2025, volume 16005 of LNCS, pages 457–479. Springer, 2025. https://doi.org/10.1007/978-3-032-01887-8_15.

[GS22] Jens Groth and Victor Shoup. On the Security of ECDSA with Additive Key Derivation and Presignatures. In Advances in Cryptology – EUROCRYPT 2022, volume 13275 of LNCS, pages 365–396. Springer, 2022. https://doi.org/10.1007/978-3-031-06944-4_13.

[PS96] David Pointcheval and Jacques Stern. Security Proofs for Signature Schemes. In Advances in Cryptology – EUROCRYPT 1996, volume 1070 of LNCS, pages 387–398. Springer, 1996. https://doi.org/10.1007/3-540-68339-9_33.